2022Call2.

(1) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$(x-1)^3 = \boxed{a} + \boxed{b}(x+1) + \boxed{c}(x+1)^2 + \boxed{d}(x+1)^3.$$

$\boxed{a}$:

NUMERICAL | 1 point

-8 ✓	

$\boxed{b}$:

NUMERICAL | 1 point

12 ✓	

$\boxed{c}$:

NUMERICAL | 1 point

-6 ✓	

$\boxed{d}$:

NUMERICAL | 1 point

1 ✓	

$$(x+1)\log(x^2) = \boxed{e} + \boxed{f}(x+1) + \boxed{g}(x+1)^2 + \boxed{h}(x+1)^3 + o((x+1)^3) \text{ as } x \to -1.$$

$\boxed{e}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{f}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{g}$:

NUMERICAL | 1 point

-2 ✓	

$\boxed{h}$:

NUMERICAL | 1 point

-1 ✓	

$\log(-x)\cdot(\exp(x+1)-1)\cdot(x+1) = \boxed{\text{i}}+\boxed{\text{j}}(x+1)+\boxed{\text{k}}(x+1)^2+\boxed{\text{l}}(x+1)^3+o((x+1)^3)$ as $x \to -1$.

$\boxed{\text{i}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{j}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{k}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{l}}$:

NUMERICAL | 1 point

-1 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x\to-1} \frac{(x-1)^3 + \alpha(x+1)\log(x^2) + 8 + \beta(x+1)}{\log(-x)\cdot(\exp(x+1)-1)\cdot(x+1)}.$$

This limit converges for $\alpha = \boxed{\text{m}}, \beta = \boxed{\text{n}}$.

$\boxed{\text{m}}$:

NUMERICAL | 4 points

-3 ✓	

$\boxed{\text{n}}$:

NUMERICAL | 4 points

-12 ✓	

In that case, the limit is $\boxed{\text{o}}$.

$\boxed{\text{o}}$:

NUMERICAL | 4 points

-4 ✓	

(2) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$(1-x)^3 = \boxed{\text{a}} + \boxed{\text{b}}(x+1) + \boxed{\text{c}}(x+1)^2 + \boxed{\text{d}}(x+1)^3.$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

8 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

-12 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 1 point

6 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

-1 ✓	

$$(x+1)\log(x^2) = \boxed{\text{e}} + \boxed{\text{f}}(x+1) + \boxed{\text{g}}(x+1)^2 + \boxed{\text{h}}(x+1)^3 + o((x+1)^3) \text{ as } x \to -1.$$

$\boxed{\text{e}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{f}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{g}}$:

NUMERICAL | 1 point

-2 ✓	

$\boxed{\text{h}}$:

NUMERICAL | 1 point

-1 ✓	

$$\log(-x)\cdot(\exp(x+1)-1)\cdot(x+1) = \boxed{\text{i}} + \boxed{\text{j}}(x+1) + \boxed{\text{k}}(x+1)^2 + \boxed{\text{l}}(x+1)^3 + o((x+1)^3) \text{ as } x \to -1.$$

$\boxed{\text{i}}$:

NUMERICAL 1 point

0 ✓	

j:

NUMERICAL 1 point

0 ✓	

k:

NUMERICAL 1 point

0 ✓	

l:

NUMERICAL 1 point

-1 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to -1} \frac{(1-x)^3 + \alpha(x+1)\log(x^2) - 8 + \beta(x+1)}{\log(-x) \cdot (\exp(x+1) - 1) \cdot (x+1)}.$$

This limit converges for $\alpha = \boxed{m}$, $\beta = \boxed{n}$.

m:

NUMERICAL 4 points

3 ✓	

n:

NUMERICAL 4 points

12 ✓	

In that case, the limit is $\boxed{o}$.

o:

NUMERICAL 4 points

4 ✓	

(3) **Q2**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Calculate the following series.

$$\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k = \frac{\boxed{a}}{\boxed{b}}.$$

a:

NUMERICAL | 1 point

4 ✓	

b:

NUMERICAL | 1 point

5 ✓	

$$\sum_{k=0}^{\infty}\left(\frac{i}{2}\right)^k = \frac{\boxed{c}}{\boxed{d}} + i\frac{\boxed{e}}{\boxed{f}}.$$

c:

NUMERICAL | 1 point

4 ✓	

d:

NUMERICAL | 1 point

5 ✓	

e:

NUMERICAL | 1 point

2 ✓	

f:

NUMERICAL | 1 point

5 ✓	

Let us study the following series $\sum_{n=0}^{\infty}\frac{x^n}{n!}$, with various x.

In order to discuss the convergence using the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{|x|^n}{n!}$. Complete the formula.

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \boxed{g}.$$

g:

NUMERICAL | 2 points

0 ✓	

Choose all the value of x for which the series converges.

MULTI | 2 points | Single

- -100 ✓
- -10 ✓
- -5 ✓
- -2 ✓
- -1 ✓
- 0 ✓

- 1 ✓
- 2 ✓
- 5 ✓
- 10 ✓
- 100 ✓

Calculate the series.

$$\sum_{n=0}^{\infty} \frac{2^n}{n!} = e^{\boxed{\text{h}}}.$$

$\boxed{\text{h}}$:

NUMERICAL | 2 points

2 ✓	

(4) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Calculate the following series.

$$\sum_{k=0}^{\infty} \left(-\frac{1}{3}\right)^k = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

4 ✓	

$$\sum_{k=0}^{\infty} \left(\frac{i}{3}\right)^k = \frac{\boxed{\text{c}}}{\boxed{\text{d}}} + i\frac{\boxed{\text{e}}}{\boxed{\text{f}}}.$$

$\boxed{\text{c}}$:

NUMERICAL | 1 point

9 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

10 ✓	

e:

NUMERICAL | 1 point

3 ✓	

f:

NUMERICAL | 1 point

10 ✓	

Let us study the following series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$, with various x.

In order to discuss the convergence using the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{|x|^n}{n!}$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \boxed{\text{g}}.$$

g:

NUMERICAL | 2 points

0 ✓	

Choose all the value of x for which the series converges.

MULTI | 2 points | Single

- -100 ✓
- -10 ✓
- -5 ✓
- -2 ✓
- -1 ✓
- 0 ✓
- 1 ✓
- 2 ✓
- 5 ✓
- 10 ✓
- 100 ✓

Calculate the series.

$$\sum_{n=0}^{\infty} \frac{3^n}{n!} = e^{\boxed{\text{h}}}.$$

h:

NUMERICAL | 2 points

3 ✓	

(5) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \exp\left(\frac{x}{x^2+1}\right).$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI | 4 points | Single

- $-e$ (-100%)
- -2 (-100%)
- -1 (-100%)
- 0 (-100%)
- 1 (-100%)
- 2 (-100%)
- e (-100%)
- All real numbers are in the domain. ✓

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single

- $y = -e$ (-100%)
- $y = -1$ (-100%)
- $y = 0$ (-100%)
- $y = 1$ ✓
- $y = e$ (-100%)
- $x = -2$ (-100%)
- $x = -\sqrt{3}$ (-100%)
- $x = -\sqrt{2}$ (-100%)
- $x = -1$ (-100%)
- $x = 0$ (-100%)
- $x = 1$ (-100%)
- $x = \sqrt{2}$ (-100%)
- $x = \sqrt{3}$ (-100%)
- $x = 2$ (-100%)
- $y = x$ (-100%)
- $y = -x$ (-100%)

One has

$$f'(2) = \exp\left(\frac{\boxed{\text{a}}}{\boxed{\text{b}}}\right)\frac{\boxed{\text{c}}}{\boxed{\text{d}}}.$$

a:

NUMERICAL | 3 points

2 ✓	

b:

NUMERICAL | 1 point

5 ✓	

c:

NUMERICAL | 3 points

-3 ✓	

d:

NUMERICAL | 1 point

25 ✓	

The function $f(x)$ has e stationary point(s):

e:

NUMERICAL | 4 points

2 ✓	

Choose the behaviour of $f(x)$ in the interval $(0, 2)$.

MULTI | 4 points | Single | Shuffle

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(6) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \exp\left(-\frac{x}{x^2+1}\right).$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI | 4 points | Single

- $-e$ (-100%)
- -2 (-100%)
- -1 (-100%)
- 0 (-100%)
- 1 (-100%)
- 2 (-100%)
- e (-100%)
- All real numbers are in the domain. ✓

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single

- $y = -e$ (-100%)
- $y = -1$ (-100%)
- $y = 0$ (-100%)
- $y = 1$ ✓
- $y = e$ (-100%)
- $x = -2$ (-100%)
- $x = -\sqrt{3}$ (-100%)
- $x = -\sqrt{2}$ (-100%)
- $x = -1$ (-100%)
- $x = 0$ (-100%)
- $x = 1$ (-100%)
- $x = \sqrt{2}$ (-100%)
- $x = \sqrt{3}$ (-100%)
- $x = 2$ (-100%)
- $y = x$ (-100%)
- $y = -x$ (-100%)

One has

$$f'(2) = \exp\left(\frac{\boxed{\text{a}}}{\boxed{\text{b}}}\right)\frac{\boxed{\text{c}}}{\boxed{\text{d}}}.$$

$\boxed{\text{a}}$:

NUMERICAL | 3 points

-2 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

5 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 3 points

3 ✓	

d:

NUMERICAL | 1 point

25 ✓	

The function $f(x)$ has e stationary point(s):

e:

NUMERICAL | 4 points

2 ✓	

Choose the behaviour of $f(x)$ in the interval $(2, 3)$.

MULTI | 4 points | Single | Shuffle

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(7) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^1 \frac{x^2}{x^3 + 2x^2 + x + 2} dx.$$

Complete the formula

$$\frac{x^2}{x^3 + 2x^2 + x + 2} = \frac{\frac{\boxed{a}}{\boxed{b}}x + \frac{\boxed{c}}{\boxed{d}}}{x^2 + \boxed{e}} + \frac{\frac{\boxed{f}}{\boxed{g}}}{x + \boxed{h}}.$$

a:

NUMERICAL | 2 points

1 ✓	

b:

NUMERICAL | 2 points

5 ✓	

c:

NUMERICAL 2 points

-2 ✓	

d:

NUMERICAL 2 points

5 ✓	

e:

NUMERICAL 2 points

1 ✓	

f:

NUMERICAL 2 points

4 ✓	

g:

NUMERICAL 2 points

5 ✓	

h:

NUMERICAL 2 points

2 ✓	

Choose a primitive of $\frac{x}{x^2+1}$.

MULTI 8 points Single Shuffle

- $\arctan(x+1)$
- $\frac{x}{2}\arctan(x)$
- $x\arctan(x^2+1)$
- $\frac{1}{4}\log(x^2+1)$
- $\frac{1}{2}\log(x^2+1)$ ✓
- $\log(x(x^2+1))$
- $\frac{1}{4}\arcsin(x^2+1)$
- $\frac{1}{2}\arcsin(x^2+1)$
- $\arcsin(x(x^2+1))$

Choose a primitive of $\frac{1}{x^2+1}$.

MULTI 8 points Single Shuffle

- $\arctan(x)$ ✓
- $\arctan(\frac{x}{2})$
- $\frac{1}{2}\arctan(\frac{x}{2})$
- $\frac{1}{4}\arctan(x)$
- $\log(x^2+1)$
- $\frac{1}{2}\log(x^2+1)$
- $x\log(x^2+1)$
- $-2x/(x^2+1)^2$

By continuing, we get

$$\int_0^1 \frac{x^2}{x^3+2x^2+x+2} = \frac{\boxed{\text{i}}}{\boxed{\text{j}}}\log 3 + \frac{\boxed{\text{m}}}{\boxed{\text{n}}}\log 2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}\pi$$

.

$\boxed{\text{i}}$:

NUMERICAL | 4 points

4 ✓	

$\boxed{\text{j}}$:

NUMERICAL | 2 points

5 ✓	

$\boxed{\text{k}}$:

NUMERICAL | 2 points

-7 ✓	

$\boxed{\text{l}}$:

NUMERICAL | 2 points

10 ✓	

$\boxed{\text{m}}$:

NUMERICAL | 4 points

-1 ✓	

$\boxed{\text{n}}$:

NUMERICAL | 2 points

10 ✓	

(8) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^1 \frac{x}{x^3+2x^2+x+2}dx.$$

Complete the formula

$$\frac{x}{x^3+2x^2+x+2} = \frac{\frac{\boxed{\text{a}}}{\boxed{\text{b}}}x + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}}{x^2+\boxed{\text{e}}} + \frac{\frac{\boxed{\text{f}}}{\boxed{\text{g}}}}{x+\boxed{\text{h}}}.$$

a:

NUMERICAL | 2 points

2 ✓	

b:

NUMERICAL | 2 points

5 ✓	

c:

NUMERICAL | 2 points

1 ✓	

d:

NUMERICAL | 2 points

5 ✓	

e:

NUMERICAL | 2 points

1 ✓	

f:

NUMERICAL | 2 points

-2 ✓	

g:

NUMERICAL | 2 points

5 ✓	

h:

NUMERICAL | 2 points

2 ✓	

Choose a primitive of $\frac{x}{x^2+1}$.

MULTI | 8 points | Single | Shuffle

- $\arctan(x+1)$
- $\frac{x}{2}\arctan(x)$
- $x\arctan(x^2+1)$
- $\frac{1}{4}\log(x^2+1)$
- $\frac{1}{2}\log(x^2+1)$ ✓
- $\log(x(x^2+1))$

- $\frac{1}{4}\arcsin(x^2+1)$
- $\frac{1}{2}\arcsin(x^2+1)$
- $\arcsin(x(x^2+1))$

Choose a primitive of $\frac{1}{x^2+1}$.

MULTI | 8 points | Single | Shuffle

- $\arctan(x)$ ✓
- $\arctan(\frac{x}{2})$
- $\frac{1}{2}\arctan(\frac{x}{2})$
- $\frac{1}{4}\arctan(x)$
- $\log(x^2+1)$
- $\frac{1}{2}\log(x^2+1)$
- $x\log(x^2+1)$
- $-2x/(x^2+1)^2$

By continuing, we get

$$\int_0^1 \frac{x}{x^3+2x^2+x+2} = \frac{\boxed{\text{i}}}{\boxed{\text{j}}}\log 3 + \frac{\boxed{\text{m}}}{\boxed{\text{n}}}\log 2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}\pi$$

.

i:

NUMERICAL | 4 points

-2 ✓	

j:

NUMERICAL | 2 points

5 ✓	

k:

NUMERICAL | 2 points

3 ✓	

l:

NUMERICAL | 2 points

5 ✓	

m:

NUMERICAL | 4 points

1 ✓	

n:

NUMERICAL | 2 points

20 ✓	

(9) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Determine whether the following improper integral converges, and if so, calculate the value. If it does not converge, write $\frac{1}{0}$.

$$\int_0^\infty x^{-\frac{1}{3}} e^{-x^{\frac{2}{3}}} dx = \frac{\boxed{a}}{\boxed{b}}.$$

$\boxed{a}$:

NUMERICAL | 6 points

3 ✓	

$\boxed{b}$:

NUMERICAL | 6 points

2 ✓	

Let us consider the improper integral $\int_1^\infty f(x)dx$. Choose all function(s) $f(x)$ for which this improper integral converges.

MULTI | 12 points | Single

- $f(x) = \exp(x)$ (−100%)
- $f(x) = \exp(-x)$ ✓
- $f(x) = x\exp(x)$ (−100%)
- $f(x) = x\exp(-x)$ ✓
- $f(x) = \frac{1}{x}\exp(x)$ (−100%)
- $f(x) = \frac{1}{x}\exp(-x)$ ✓
- $f(x) = \exp(x^2)$ (−100%)
- $f(x) = \exp(-x^2)$ ✓
- $f(x) = x^2\exp(x^2)$ (−100%)
- $f(x) = x^2\exp(-x^2)$ ✓
- $f(x) = \frac{1}{x}\exp(x^2)$ (−100%)
- $f(x) = \frac{1}{x}\exp(-x^2)$ ✓

Among the following improper integrals, choose the smallest (and convergent) one and give its value $\frac{\boxed{c}}{\boxed{d}} e^{\boxed{e}}$.

- $\int_0^\infty \exp(-x)$
- $\int_0^\infty x\exp(-x)dx$
- $\int_0^\infty x^2\exp(-x)dx$
- $\int_0^\infty x\exp(-3x)dx$

- $\int_0^\infty x^2 \exp(-3x)dx$
- $\int_1^\infty \exp(-3x)$
- $\int_1^\infty \exp(-4x)$

c:

NUMERICAL | 2 points

1 ✓	

d:

NUMERICAL | 5 points

4 ✓	

e:

NUMERICAL | 5 points

-4 ✓	

(10) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Determine whether the following improper integral converges, and if so, calculate the value. If it does not converge, write $\frac{1}{0}$.

$$\int_0^\infty x^{-\frac{1}{4}} e^{-x^{\frac{3}{4}}} dx = \frac{\boxed{a}}{\boxed{b}}.$$

a:

NUMERICAL | 6 points

4 ✓	

b:

NUMERICAL | 6 points

3 ✓	

Let us consider the improper integral $\int_1^\infty f(x)dx$. Choose all function(s) $f(x)$ for which this improper integral converges.

MULTI | 12 points | Single

- $f(x) = \exp(x)$ (−100%)
- $f(x) = \exp(-x)$ ✓
- $f(x) = x\exp(x)$ (−100%)
- $f(x) = x\exp(-x)$ ✓

- $f(x) = \frac{1}{x}\exp(x)$ (-100%)
- $f(x) = \frac{1}{x}\exp(-x)$ ✓
- $f(x) = \exp(x^2)$ (-100%)
- $f(x) = \exp(-x^2)$ ✓
- $f(x) = x^2\exp(x^2)$ (-100%)
- $f(x) = x^2\exp(-x^2)$ ✓
- $f(x) = \frac{1}{x}\exp(x^2)$ (-100%)
- $f(x) = \frac{1}{x}\exp(-x^2)$ ✓

Among the following improper integrals, choose the smallest (and convergent) one and give its value $\frac{\boxed{c}}{\boxed{d}}e^{\boxed{e}}$.

- $\int_0^\infty \exp(-x)$
- $\int_0^\infty x\exp(-x)dx$
- $\int_0^\infty x^2\exp(-x)dx$
- $\int_0^\infty x\exp(-3x)dx$
- $\int_0^\infty x^2\exp(-3x)dx$
- $\int_1^\infty \exp(-3x)$
- $\int_1^\infty \exp(-5x)$

c:

NUMERICAL | 2 points

1 ✓	

d:

NUMERICAL | 5 points

5 ✓	

e:

NUMERICAL | 5 points

-5 ✓	

Total of marks: 288