BSc Engineering Sciences – A. Y. 2017/18

**Written exam of the course Mathematical Analysis 2**

September 17, 2018

Last name: ................................................ First name: .......................................................

Matriculation: ................................................

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Solve the following problems, motivating in detail the answers.

**1.** Study the conditional, absolute and uniform convergence of the series

$$\sum_{n=0}^{+\infty} \frac{1}{3^n}(\sqrt{n+1} - \sqrt{n})(2x^2 - 5)^n \ .$$

*Solution.*

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**2.** Find the extremal values of the function $f(x, y) = 3x - 4y$ on the curve $C$ defined by $3x^2 + 2y^2 = 1$.
*Solution.*

Matriculation: ...............................................

**3.** Determine whether the following vector field on $\mathbb{R}^2$

$$\mathbb{f}(x,y) = \left( \frac{e^x}{e^x + y^2}, \frac{2y}{e^x + y^2} \right)$$

is a gradient of some scalar field. If so, find one of these scalar fields $\varphi$ such that $\mathbb{f}(x,y) = \nabla\varphi(x,y)$.

*Solution.*

Matriculation: ................................................

**4.** Compute the double integral

$$\iint_T y\left[x\log(1+\sqrt{y})+\frac{1}{1+x^2}\right]dxdy,$$

where $T=\{(x,y)\in\mathbb{R}^2\,:\,x^2+y^2\leq 1,\;y\geq 0\}$.
*Solution.*

Matriculation: ..............................................

**5.** Let $\mathbb{F}(x, y, z) = (xy^2, xz^2, y^2z)$ be a vector field on $\mathbb{R}^3$, $S$ be the surface of the cylinder:

$$S := \{(x, y, z) : 0 \leq x^2 + y^2 \leq 1, 0 \leq z \leq 2\},$$

and $\mathbb{n}$ the outgoing normal unit vector on $S$ at each point of $S$.

Compute the surface integral

$$\iint\limits_S \mathbb{F} \cdot \mathbb{n} \, dS.$$

*Solution.*