Constructive QFT and formal mathematics

2024. 05. 23 Erlangen

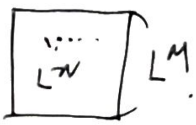
1. (Mathematical) Quantum Field Theory.

QFT: physical theory of elementary particles (standard model, QED, QCD..)
General frameworks: Wightman, Osterwalder-Schrader, Araki-Haag-Kastler.
No <u>realistic</u> example in $d=3+1$.
Constructing Yang-Mills theory is a Millenium probem.
The best results: Balaban ~ 1989.
Constructive QFT: trying to construct examples.
Glimm-Jaffe, Lechner, Adamo-Giorgetti-T. $d=1+1$, $2+1$.

2. OS axioms.

$\{S_n\}$: set of n-point "functions" (distributions on $\mathbb{R}^{nd}$) satisfying
Regularity, Euclidean invariance, Reflection positivity, symmetry, Clustering.
Considered as a model of QFT ($\Rightarrow$ Wightman field).

3. Lattice construction (of the ϕ^4_d-theory).



Discretize $\mathbb{R}^d$ to $\mathbb{T}_M^{-N} = L^{-N}\mathbb{Z}^d / L^M \mathbb{Z}^d$
Field $\phi: \mathbb{T}_M^{-N} \to \mathbb{R}$
Action $S^N(\phi) = \frac{1}{L^{Nd}} \sum_{x\in\mathbb{T}_M^{-N}} \frac{1}{2}\sum_{\mu}^{d} \partial\phi(x)^2 + \frac{\mu_N}{2}\phi(x)^2 + \frac{\lambda_N}{4}\phi(x)^4 + \varepsilon_N$
with some parameters μ_N, λ_N, ε_N.
Partition function (normalization constant) $Z^N = \int \exp(-S^N(\phi))\mathcal{D}\phi$ $\quad \mathcal{D}\phi = \prod_{x\in\mathbb{T}_M^{-N}} d\phi(x)$.
$\exp(-S(\phi))/Z^N$ defines a probability distribution on $\mathbb{R}^{\mathbb{T}_M^{-N}}$
n-point functions $G^N(x_1, .. x_n) = \int \phi(x_1) .. \phi(x_n) \exp(-S^N(\phi))\mathcal{D}\phi / Z^N$.
Convergent for nice μ_N, λ_N, ε_N? OS axioms?
"UV stability" if Z^N/Z^N_{free} is bounded.
Balaban: UV stability for the Yang-Mills theory.

4. Formalization?

Balaban, King's papers.. (cf. Dimock 2011~, Dybalski-Stottmeister-T.)
Are new results (of ours) OK?? Who can tell?
Perfectoid space (Scholze) in Lean (Buzzard-Commelin-Massot 2020)
Polynomial Freiman-Ruzsa conjecture (Gowers-Green-Manners-Tao) in Lean (2023)
Next things in Mathlib?